Induction Plasmas at Low Frequencies

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The possibility of generating thermal electrodeless plasmas at frequencies below the rf range through the use of magnetic materials has been investigated, both analytically and experimentally. Existing minimum frequency and voltage criteria for the maintenance of quasi-steady, tube filling a.c. arcs were combined with the induction law to yield design criteria for a "plasma transformer" which consists of a conventional iron core and primary but has the secondary replaced by a torus-shaped vessel containing a single plasma turn. The idea has been tested, at a frequency of 9600 Hz, on a small model with an iron core of 100 cm² cross section. Glow and arc discharges at pressures up to 600 torr have been generated in argon. Applicability to operation at 60 Hz is discussed.

I. Introduction

THE development of heaters for hypersonic tunnels, chemical reactors, and other large scale devices involving applications of thermal plasmas would be greatly advanced if there were a more economical method for generating such plasmas than is presently available. Heating by a direct current arc, the most common method, involves up to 50% electrode cooling losses and, moreover, leads to short electrode life and plasma contamination by chemically active gases. Heating by magnetic induction, the alternate method, avoids the disadvantages of electrodes but, as a literature survey indicates, is presently carried out only at frequencies in the megacycle range. To generate such frequencies requires expensive vacuum tube oscillators in addition to the power conditioning equipment used with d.c. heaters. On the other hand it has been shown[1–3] that it is possible to maintain induction plasmas in the glow discharge regime using power obtained directly from motor-generators operating at audio frequencies, if a magnetic core is used to improve the coupling. Smith[1] in 1941 reported a ring discharge at 900 Hz in Hg-vapor of 10^{-4} torr pressure. The discharge vessel had toroidal shape and a volume of several gallons. It was placed over a straight, laminated iron core of several meters length. Ionization was initiated by a small hot cathode discharge. Barger, Brooks, and Beasley[2,3] used the magnetic field gradient along an iron rod excited at one end by a 9600 Hz current for continuous acceleration of argon plasmas at pressures below on torr. An rf excited coil helped to start the discharge.

If it were feasible to generate thermal plasmas in the same manner, and at the same time to lower the frequency further into the range of power line frequencies, the conversion of electric power into plasma heat would be greatly facilitated. By using a fully closed iron path, as in a conventional transformer, one would then be able to attain higher conversion efficiencies than are possible with either the d.c. or the rf method.

The purpose of the present paper is twofold, first, to establish scaling relationships from which the dimensions of a "plasma transformer" operating at line or audiofrequencies can be deduced and, second, to describe an experimental investigation which showed that, with a closed magnetic circuit, thermal induction plasmas can be generated at audiofrequencies and power levels comparable to those in the quoted glow discharge experiments.[1–3]

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II. Analysis

An induction plasma can be maintained for an extended time only by a sequence of voltage pulses, as the change in magnetic flux, which generates the voltage, is time limited. Such a plasma must therefore have a pulsating character. If the time interval between pulses is small compared to the decay time of the plasma, as it is generally the case with rf excitation, the pulsations are negligible and the rms value of the induced voltage equals the maintenance voltage of a steady-state plasma. However, at lower frequencies the situation is not so simple; one can apply steady-state analysis only if the product of decay time τ and angular frequency ω is sufficiently large. This problem has been investigated by Edels and Fenlon[4] in an analysis of tube filling a.c. arc columns. Some of their results are, for convenience, reproduced in Fig. 1, which shows the relative variations of the maintenance field E and of the conductance G over one period, for different values of $\omega\tau$. It is seen that for $\omega\tau = 10$, E is practically sinusoidal and G shows only small fluctuations around the steady-state value. We will therefore, in the following discussion, require that

$$\omega\tau \geq 10 \tag{1}$$

and refer to Eq. (1) as the continuity criterion. For a glow discharge column controlled by electron diffusion, τ is given by the relation

$$\tau = \Lambda^2/D_a \tag{2}$$

where Λ, the diffusion length, is equal to r/λ for a column with circular cross section of radius r, D_a is the ambipolar diffusion coefficient, and λ is the first zero of the Bessel function. It has the value 2.405 if the electron density vanishes or the temperature is zero at the wall. For a thermal plasma column, controlled by heat conduction losses to the tube walls, D_a is replaced by the thermal diffusivity $K = \kappa/c_p\rho$ (κ = heat conductivity, c_p = specific heat at constant pressure, ρ = mass density). Equation (2) then becomes

$$\tau = \Lambda^2/K \tag{2a}$$

For a nitrogen[5] or argon[6] column at atmospheric pressure and a temperature of $10^4 K$, $K \approx 200$ cm²/sec and therefore $\tau \gtrsim 10^{-3} r^2$ sec/cm². The maintenance criterion (1) becomes in this case

$$\omega r^2 \gtrsim 10^4 \text{cm}^2/\text{sec} \tag{3}$$

At $f = 60$ Hz, for example, it is necessary that $r \gtrsim 5$ cm. If Eq. (1) is satisfied, the induction plasma behaves approximately like that of a d.c. discharge and can be maintained by an alternating voltage whose rms value equals the d.c. maintenance voltage. For a tube filling arc of length l, the maintenance voltage is, according to Edels and Fenlon,[4]

$$V_m = El = \lambda l/(B^{1/2}r) \tag{4}$$

where B is a constant (with dimension volts^{-2}), which for argon at atmospheric pressure[6] has the value of about 1.6. Thus $\lambda/B^{1/2} \approx 2$ v. According to Eq. (4) the electric field should be inversely proportional to the arc radius and independent of the arc current. To check the validity of this relationship, experimental data on the characteristics of confined argon arcs from the work of Emmons[7] have been replotted in Fig. 2. One notices that for the small radius $r = 0.5$ cm Eq. (4) holds only at the minimum, for $r = 1$ cm the dependence on the current is much lower, and for $r = 2$ cm it is practically zero within the range plotted. One may thus infer that Eq. (4) will give a fair representation for cases where $r \gtrsim 2$ cm; this is also the regime to which we are confined by Eq. (3) for very low frequencies ($f < 400$ Hz).

The voltage induced by a magnetic flux which is confined to a circular area πR^2 by a material of effective permeability μ and which alternates at angular frequency ω is given by the relation

$$|V_i| = \pi R^2 \mu\omega H \tag{5}$$

where H is the magnetic excitation in amp/cm. Since there is a practical limit for H and since the maintenance voltage is independent of frequency as long as Eq. (1) is satisfied, it is seen from Eq. (5) that a tradeoff between μ and ω is possible. If, for example, iron ($\mu \sim 10^3\mu_o$) is used for the magnetic path, the frequency required, in order that the induced voltage V_i be great enough to maintain the plasma, is reduced from the megahertz regime to the kilohertz regime. To generate power at these frequencies rotating machinery can be used and such intermediate power conditioning equipment as high-voltage transformers, rectifiers, and vacuum tube oscillators is no longer needed. If the flux area is increased, the same voltage can be generated at even lower frequencies, and once the lower desirable limit of 60 Hz has been reached, further increases in area can be used to produce larger values of V_i.

We now imagine the discharge tube to be bent into a circle and its ends connected to form a torus which tightly fits over the magnetic core, so that the average path length for the current will be

$$l = 2\pi(R + r) \tag{6}$$

Introducing this value for l into Eq. (4) and setting,

$$V_m = V_i \tag{7}$$

which we will refer to as the maintenance criterion, we obtain a quadratic expression for the ratio R/r. The solution is

$$R/r = a[1 + (1 + 2/a)^{1/2}] \tag{8}$$

where

$$a = \lambda/B^{1/2}\omega r^2 \mu H \tag{9}$$

If we now introduce the specialized continuity criterion Eq. (3) into Eq. (9) and use the numerical values for λ and B, we obtain

$$a \approx 2 \times 10^{-4}/\mu H \tag{10}$$

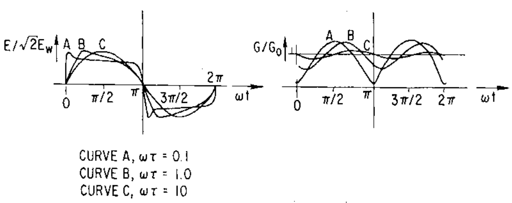

Fig. 1 Field and conductance waveforms of a.c. arcs for various $\omega\tau$ after Edels and Fenlon.[4]

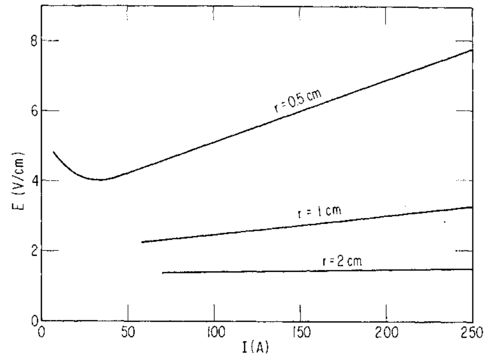

Fig. 2 Characteristics of confined argon arcs after Emmons[7]; $p = 1$ atm, r = tube radius.

where μH, the magnetic flux density, is measured in v sec/cm^2 Since 1 v sec/cm^2 = 10^8 gauss Eq. (10) can also be written

$$a = 2 \times 10^4/\mu H\,[\text{gauss}] \tag{10a}$$

Since ω does not appear in Eqs. (10) or (10a), the ratio R/r in Eq. (8) is independent of frequency. Equation (8) has been evaluated with the aid of Eq. (10a), and from the R/r ratios so obtained along with values for r from Eq. (3), absolute values for R have been calculated. Both R and r have been plotted in Fig. 3 over a frequency range from 10 to 10^4 Hz. The parameter for R in the figure is the flux density μH. The maximum flux density obtainable for continuous operation at the low-frequency end is about 15,000 gauss, whereas it is 5000 gauss at the high-frequency end. The minimum core radius required at $f = 9600$ Hz, a common generator frequency, will therefore be 3.5 cm, whereas for 60 Hz and 15,000 gauss, it will be 18 cm. The line at $f = 180$ Hz in the figure was drawn because this frequency has practical significance; it can be generated by static frequency multipliers which convert 3 phase 60 Hz power into single phase power. The advantage of these devices is that uniform phase loading is made possible, and that the transformer can be scaled down by a factor of $3^{\frac{1}{2}}$ in comparison with one operating at 60 Hz.

It is of interest also to calculate the power dissipated in the plasma. The power lost per centimeter column length in a tube filling arc is[4]

$$P/l = 2\pi\lambda S_0 J_1(\lambda) \tag{11}$$

where S_o is the heat conduction potential at the tube axis and $J_1(\lambda) = 0.519$. For argon at $T = 10^4$°K and $p = 1$ atm $S_o = 20.6$ w/cm.[7] Using numerical values and Eq. (6) we can write Eq. (11) in the form

$$P = 1.02(R + r)\,[kw] \tag{12}$$

For a constant axial temperature, P is thus proportional to the sum of the two radii. A plot of P vs frequency for different

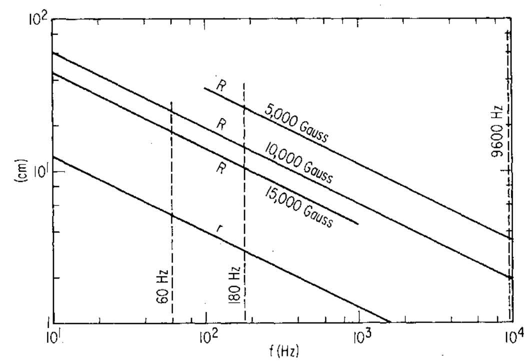

Fig. 3 Minimum values of magnet core radius R and tube radius r required to maintain electrodeless arc in argon at 1 atm; parameter: magnetic flux density.

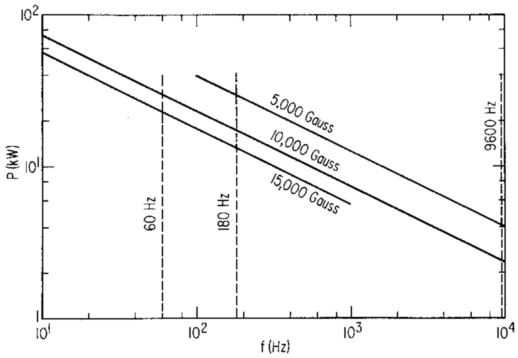

Fig. 4 Minimum power required for electrodeless arc in argon at $p = 1$ atm and various magnetic flux densities.

flux densities is shown in Fig. 4. For a frequency of 60 Hz and a flux density of 15,000 gauss, P is seen to be 23 kw. At 9600 Hz and 5000 gauss, it is 4 kw. It should be kept in mind that this calculation takes into account only conduction losses to the tube walls. Losses by radiation and convection will increase the amount of absorbed power.

At high-power densities the column may contract and the maintenance field become more or less independent of the tube radius and the preceding analysis will no longer apply. The maintenance voltage V_m will than be approximately proportional to R. Since the induced voltage V_i grows with R^2, however, there will still exist a value of R for which $V_m = V_i$.

III. Experiment

The feasibility of maintaining an electrodeless thermal plasma at audiofrequencies was tested in an arrangement shown schematically in Fig. 5. The power source was a three phase motor-generator of variable output and a constant frequency of 9600 Hz.† At full output the rms voltage per phase was 440 v. The terminals of three independent generator phases were connected in series to obtain a maximum voltage of 880 v. In series with the five-turn primary of the transformer was a capacitor of about 0.5 μF which served as a current limiter upon breakdown of the gas. The magnetic path consisted of four C-shaped supersil cores with a lamination thickness of 0.1 mm and 5 × 10 cm² cross section which were arranged in 00 (i.e., double 0) fashion. The primary current I_1 and the induced voltage per turn V_2 were measured directly as shown. To obtain information also about I_2, the secondary current in the plasma, the transformer was calibrated with a variable resistive load. The result was the set of curves of Fig. 6, from which I_2 can be read for known values of I_1 and V_2. Since the lamination is rather coarse for the frequency used, the core losses are high, and as the figure shows, appreciable primary currents flow even in absence of a load in the secondary circuit ($I_2 = 0$).

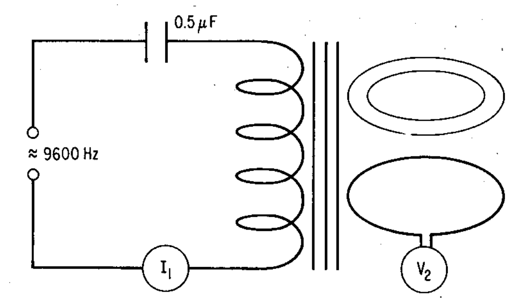

Fig. 5 Schematic of audiofrequency induction plasma experiment.

† The same generator had been used in the experiments of Refs. 2 and 3.

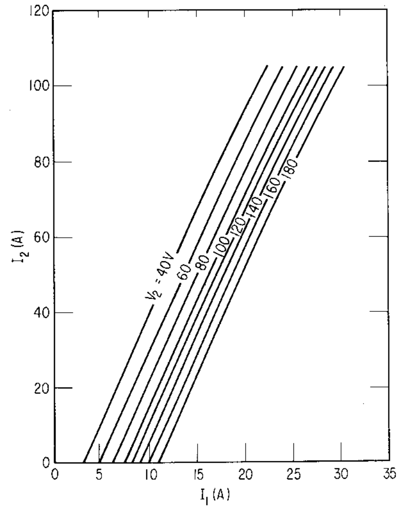

Fig. 6 Secondary current I_2 vs primary current I_1 in 9600 Hz plasma transformer; parameter: induced voltage per turn V_2.

The discharge vessel consisted of a quartz toroid of about 20 cm² cross section and a path length at the center line of 65 cm. The vessel was connected to a vacuum pump and an argon gas supply which were so regulated that the pressure stayed around 0.1 torr. To facilitate breakdown, auxiliary electrodes, made of 2 mm tungsten wires, were mounted in necks on opposite ends of the vessel and connected to the terminals of a neon sign transformer. Prebreakdown, visible as a weak glow, required about 1200 v between the electrodes. If then the generator control was turned to full output, the induced field of about 3 v/cm was sufficient to complete the breakdown and the entire torus lighted up as shown in Fig. 7. The color usually changed within seconds from a bright red to a pale blue. The preionization voltage could then be turned off without visibly affecting the discharge. The electrodes were mounted far enough back in the necks of the torus to be out of contact with the plasma when not energized. After a few firings the vessel was sufficiently baked out to require no longer preionization.

The maintenance voltage of an established glow discharge was only 15–20 v, corresponding to field strengths around 0.25 v/cm, i.e., less than 10% of the value required for breakdown. Maximum primary current was 30 amp which according to Fig. 6 indicates plasma currents well above 100 amp.

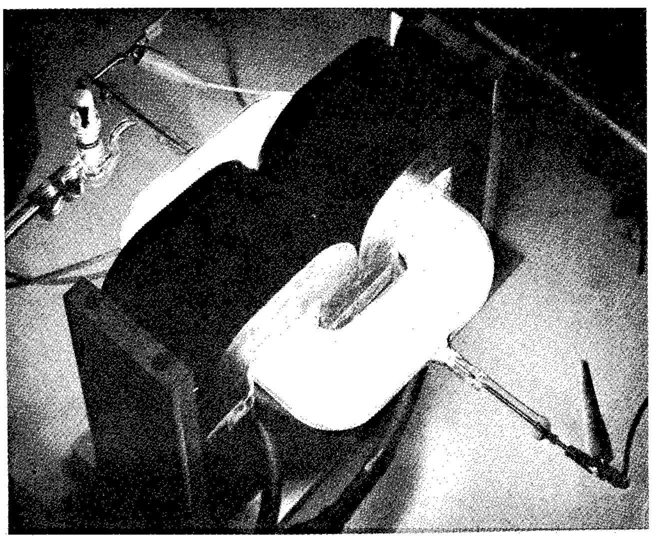

Fig. 7 Induction glow discharge at 9600 Hz; $p \approx 0.1$ torr, $V_2 \sim 15$ v, $I_2 \sim 150$ amp.

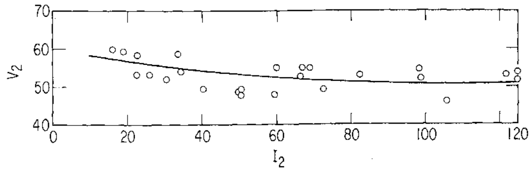

Fig. 8 Voltage-current characteristics of 9600 Hz induction plasma in argon at intermediate pressure; $p \sim 50$ torr.

It was found that after establishment of a discharge the pressure could be raised substantially. Between 10 and 20 torr the plasma turned whitish and took on a more arclike appearance. Heat output increased considerably and required cooling of vessel and core by several fans and air blowers. A plot of voltage-current data from a discharge in this intermediate pressure range ($p \sim 50$ torr) is shown in Fig. 8. Over a current range from 15 to 120 amp the data indicate a slightly negative characteristic. The maintenance field is here around 0.8 v/cm.

Upon further pressure rise the column became more brilliant and began to separate from the wall and to wobble. At a pressure of about 400 torr the column had contracted to a filament of about 2 cm thickness, as shown in Fig. 9, and the oscillations had become so violent that the strikes against the tube wall were at times audible above the noise of the motor-generator. The induced voltage at this pressure was measured to be 120 v, corresponding to a field of nearly 2 v/cm. For a primary current $I_1 = 28$ amp, Fig. 6 gives for the plasma current $I_2 \approx 105$ amp. At pressures around 500 torr the column extinguished, partly because of the lack of a stabilizing mechanism. It was found that the pressure limit could be extended well above 600 torr by superimposing some circulation on the gas. This was done by using separate ports, about 20 cm apart, for gas injection and pumping.

No temperature measurements were made on the plasma but a rough estimate of the temperature level was obtained in the following way. On the assumption that the inductive component of V_2 can be disregarded in comparison with the resistive component, it follows that the resistance of the plasma column $\mathcal{R}$ at $p = 400$ torr is 120 v/105 amp = 1.15Ω. For a column length of 65 cm and a column radius of 1 cm, one finds for the mean conductivity of the plasma, $\bar{\sigma} = l/\mathcal{R}q = 18$ mho/cm. This value corresponds to equilibrium temperatures between 8500 and 9000°K.‡ Thus the temperature level is about the same as in rf induction plasmas of similar radii.[8] Support for the assumption of a dominantly resistive voltage in the plasma can be drawn from the observations of Dymshits and Koretski[9] in rf discharges and also from the fact that the measured maintenance field of 2 v/cm agrees with the theoretical value from Eq. (4).

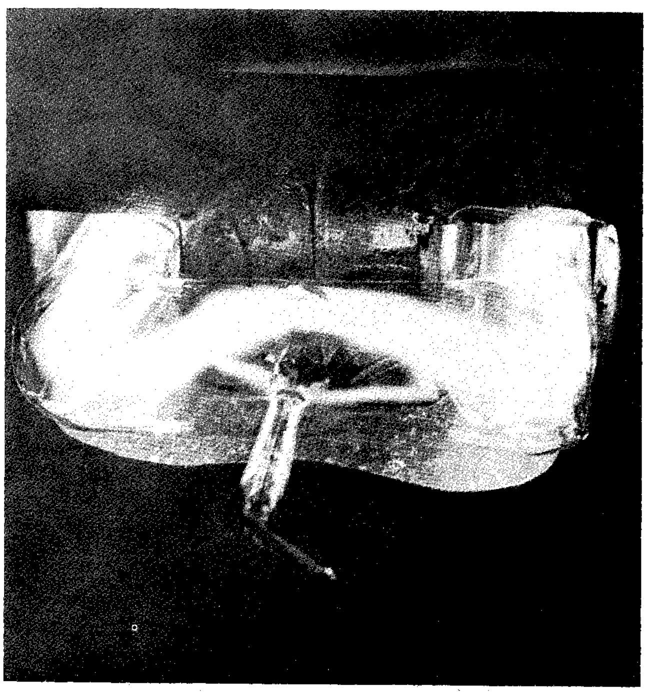

Fig. 9 Inductive electrodeless arc in argon at 9600 Hz; $p \sim 400$ torr, $V_2 \approx 120$ v, $I_2 \sim 105$ amp.

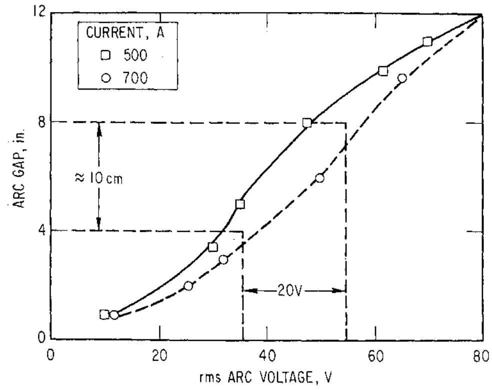

Fig. 10 Flow-stabilized a.c. arc in argon after Lanzo.[10] Arc voltage as function of arc gap; pressure: 400 torr.

Arclike discharges at pressures around 100 torr could also be maintained in helium and air.

IV. Discussion

The separation of the arc column from the tube wall is believed to be the reason why operation at atmospheric pressure, although closely reached, was not feasible with the aforementioned device. The cause for this separation is not known. While it is well known that arc columns may contract due to thermal or magnetic effects, it may in the present case also have been caused by saturation of the magnetic core and competing eddy currents. A larger core with finer lamination would be required to investigate this possibility.

To obtain an estimate of the core size and power requirements also for a separated arc at atmospheric pressure, the data by Lanzo[10] on flow stabilized a.c. argon arcs have been used which are reproduced in Figs. 10 and 11. From Fig. 10 it can be seen that for currents between 500 and 700 amp and a pressure of 8 psia (≈410 torr) the maintenance voltage increases by about 5 v for an increase in gap width by one in. The maintenance field is thus about 2 v/cm which agrees with the value measured for the induction arc at the same pressure but with only 20% of the current. Fig. 11 shows that for pressures up to 15 psia the arc voltage is nearly proportional to pressure so that for one atmosphere we can assume $E \approx$

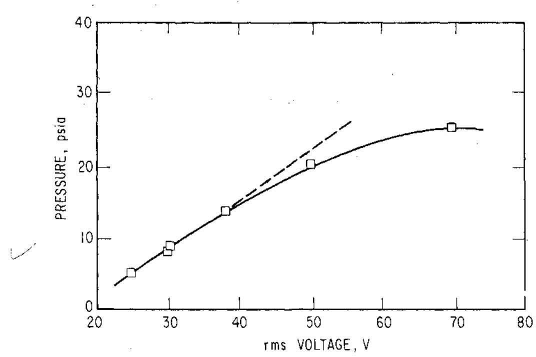

Fig. 11 Flow-stabilized a.c. arc in argon after Lanzo[10]; relationship between arc voltage and static pressure; arc gap 7.62 cm, current 550 amp.

‡ From private communication with S. Devoto, Stanford Univ.

4 v/cm. From $E = \pi f \mu H (R + r)$ we obtain by setting $r \ll R$ and $\mu H = 15{,}000$ gauss, $R = 140$ cm for $f = 60$ Hz, and $R = 47$ cm for $f = 180$ Hz. It is seen that in this case, where we can consider E as independent of r, the effect of frequency conversion is even more beneficial since R may be reduced in direct proportion to f.

The dissipated power is $P = 2\pi REI$. With $I = 500$ amp we obtain

$$P = 1760 \text{ kw for } f = 60 \text{ Hz}$$

and

$$P = 590 \text{ kw for } f = 180 \text{ Hz}$$

For operation with nitrogen or air where maintenance fields are several times higher than in argon[11] minimum core radii at 180 Hz can be expected to be in the meter range and power levels in the megawatt range unless one wants to operate below atmospheric pressure. For windtunnel heaters such power levels are not uncommon. It is only with air and other reactive gases that the advantages of the electrodeless heating become important and only at these power levels that the installation and operating costs of rf heaters become prohibitive.

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Dynamic Equilibrium of a Compound Pendulum in an Artificial Satellite

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The equation of motion of a rigid body which is constrained to rotate about an axis fixed in an artificial satellite is presented, and the stability of particular solutions of this equation is studied both for Earth-pointing and rotating satellites. Two illustrative examples indicate possible uses of such a system as a device for establishing an attitude reference or for detecting variations in satellite rotation rates.

1. Introduction

IN a paper[1] read before the Royal Irish Academy only about one year after the launching of the first artificial satellite of the Earth, J. L. Synge discussed the behavior of a pendulum attached to such a satellite, taking the pendulum to be a particle fastened to the mass center of the satellite by means of a light rod and a universal joint. In this analysis, the effect of the pendulum on the motion of the satellite was presumed to be negligible, and it is this presumption that distinguishes both Synge's paper and the present one from the many that have dealt in the intervening years with pendulum-like devices in orbit. The difference between our work and that of Synge is that we take the pendulum to be a rigid body, rather than a particle, and constrain this body to rotate about an axis fixed in the satellite in an arbitrary position.

The system under consideration is described in detail in Sec. 2. In Sec. 3, the equation of motion is presented, and the stability of particular solutions of this equation is then analyzed in Sec. 4, which also contains illustrative examples intended to point out possible practical applications. Finally, friction effects are discussed briefly in Sec. 5.

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2. System Description

The system to be analyzed is shown schematically in Fig. 1, where O designates a particle fixed in an inertial reference frame, B is an artificial satellite whose mass center, Q, moves in a circular orbit centered at O, and C is a rigid body (com-